

# Microsimulation and Income

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## Abstract

A conceptual discussion of microsimulation models in economics, using tax-benefit microsimulation as a case-study.

## 1 What is Microsimulation?

When it comes to building a model to answer a question, different techniques offer different advantages with regards to certain elements of the question at hand.

In a general sense, all economic modelling involves the construction of counterfactual worlds, which allows us to deduce conclusions about this fictional world. A model can be seen as useful for policy analysis if it is *credible* in the sense that we can use data and this counterfactual scenario to inductively reach conclusions about the real world [41].

When it comes to looking at the impact of policy change Spadaro (2007)[40] discusses three outcomes we value from a model:

- Simplicity of use and interpretation
- An ability to describe the complexity of the socioeconomic structure
- The ability to capture the heterogeneity of agents

However, there is no one model that is dominant over all three outcomes, and as a result modellers face a trade-off between these outcomes when picking a modelling technique. Typically *representative agent* models focus on the first outcome, while models that ignore the first element too much (thereby giving results that are hard to explain or causally link) tend to be termed *black box* approaches to analysis.

While a representative agent model limits the degree of heterogeneity in modelled agents, it is both clear to interpret and is able to incorporate substantial behavioural elements. In some sense, this allows modellers to answer the *Lucas Critique* [28] when observing the impact of policy.

However, by excluding heterogeneity this approach is unable to fully deal with the Lucas Critique - as the heterogeneity of agents in terms of their characteris-

tics (and the behavioural responses this entails), is policy relevant information that influences the impact of policy<sup>1</sup>.

In truth the Lucas Critique is really stating that we need appropriate behavioural structure in order to estimate the impact of policy through the lens of historic data. As a result when trying to look at the impact of policy, we want to pick a modelling approach (or approaches) that allows us to maximise this.

A microsimulation model requires three broad elements for its construction [40]:

- A microdata set containing the economic and sociodemographic characteristics of individual agents.
- An institutional framework. This is the rules of the policies to be simulated (eg budget constraints).
- A theoretical model representing the behaviour of agents.

A microsimulation model takes microdata and rules/policy, then applies the behaviour of individuals in order to simulate the economy as a function of policy. Given this, it is possible to discuss the impact of a policy based on the different characteristics of individual agents - where the heterogeneity of individual agents is described through difference in the agents measured characteristics.

## 2 Tax-Benefit policy analysis: An example of microsimulation

### 2.1 Tax-Benefit Arithmetic Microsimulation

As an example, we can ask the following policy question: what is the aggregate and distributional outcome of a change in tax-benefit policies in terms of government revenue and the income of households/agents.

We will focus on this as a *positive* question of comparing and describing the outcomes of tax-benefit policies - not as a *normative* question of what tax-benefit system is the best. Furthermore, we will leave the definition of the agent abstract, rather than picking the individual, household, family, or equivalised household.

In this context, a representative agent approach to this question is insufficient for two reasons: There is no scope for distributional analysis with a single type

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<sup>1</sup>Looking at the macroeconomy, representative agent models rose in popularity out of criticism of large-scale econometric models on the basis of the mathematical/logical link between aggregate variables. In a similar way, the mathematical link between the primitives of individuals and macro-aggregates, is often not available or cannot be defined. This is discussed in Orcutt (1957)[37], Goldman and Uzawa (1964)[15], and Shafer and Sonnenschein (1982)[39] with regards to individuals to macro theory, Mas-Colell (1989)[33] for the capital controversy, and Hoover (2001)[20] for the relation between macro and micro economics. A series of related essays can be found in Hahn and Petri (2004)[17]. The key point in what we are discussing is that the behaviour of groups can often not be sufficiently described as a mathematical function of the behaviour of a representative individual for many questions of interest.

of agent, and differing responses by heterogeneous agents are sizable enough to impact upon the aggregate outcome<sup>2</sup>.

An arithmetic microsimulation model (AMSM) offers the complete opposite approach to a representative agent model for looking at the question of comparing the outcomes of differing tax-benefit policies. A representative agent model takes a single agent, but includes behavioural responses to changes in tax and benefit policy. An AMSM on the other hand models a huge variety of agents with different characteristics, but with no behavioural response to the change in the tax benefit system. We can think of a simulation involving no behavioural response as a model assuming that behaviour will not change due to policy. This provides a clear simplifying assumption [32].

When looking at household income, an AMSM keeps gross wages fixed and keeps household labour supply behaviour unchanged as the net wage changes. As a result, changes in the tax-benefit system change net income solely in a direct *first order* manner - the changes in net income are linear in policy. So here the trade-off between a representative agent model and a AMSM is clear:

- The representative agent model offers a *behavioural* response to changes in policy including an estimate of changes in aggregate labour supply and general equilibrium effects. An AMSM does neither.
- The representative agent model treats all individuals in society as homogeneous. An AMSM uses data on the heterogeneity of individuals to loosen this assumption.

## 2.2 Adding agent behaviour: Labour Supply and Behavioural Microsimulation

Although a AMSM can provide useful results, especially for marginal changes to the tax-benefit system, the lack of any behavioural responses by agents is problematic. This issue becomes more acute the larger the changes to the tax-benefit system we are analysing.

A behavioural microsimulation model (BMSM) loosens the assumption that agents do not respond to changes in policy. Agents can now change their hours of work as their net wage changes. A BMSM does this by using household micro-data to generate the behaviour of agents as a function of policy, through the estimation of a structural econometric model and/or the calibration of a model with a given structure. The first model of this type was Hausman (1980)[18].

However, in the most common BMSMs there are still a number of simplifying assumptions included that are loosened in different forms of representative agent models:

- The choice made by agents is *static*. Namely the stock of characteristics and the choice of work vs leisure is given without reference to choices in the past or future.

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<sup>2</sup>Eg the progressivity of the tax system implies we need to know about the distribution as well as the mean for thinking about the revenue from a tax change.

- General equilibrium effects are missing. Specifically gross wages remain unchanged, relative wage rates and relative goods prices also don't change.

The behavioural simulation uses agents that aim to maximise their utility, which is a positive function of consumption,  $c$ , and leisure,  $l$ . Households are endowed with their characteristics  $z$ , a quantity of hours  $TE$ , and some non-labour income  $y_0$ . They then have the choice of spending some of their hours on leisure, or working to generate income they can spend on consumption. As a result, their choice variable is their hours of work,  $h$ .

Defining the vector of gross wages<sup>3</sup> as  $w$  and net taxes as  $NT$ , the agents maximisation problem takes the form:

$$\text{Max } u(c, l; z; \beta, \epsilon) \text{ st } c \leq y_0 + wh - NT(wh, h, y_0; z; \gamma) \quad (1)$$

Where  $\gamma$  represents parameters of the tax-benefit system, and  $\beta$  and  $\epsilon$  are coefficients that parameterise preferences [40].

The maximisation yields a labour supply function that we wish to estimate from the household microdata:

$$h = F(w, y_0; z; \beta, \epsilon; \gamma) \quad (2)$$

Where  $\beta$  and  $\epsilon$  need to be estimated, as everything else is observed. For each agent,  $i$ ,  $\beta$  is defined as a shared preference term, and  $\epsilon_i$  is an idiosyncratic individual preference (treated as a random error term). Given this, for each agent we estimate:

$$h_i = F(z_i, w_i, y_{0i}; \beta, \epsilon_i; \gamma) \quad (3)$$

Given this structure, we can simulate different tax-benefit policies by adjusting  $\gamma$ .

However, MacCurdy et al. (1990)[29] raise significant concerns about these types of models. Given non-linear taxes the data will give a non-linear budget constraint, and it is even possible for parts of the budget constraint to be non-convex. In order to get *model coherency* it is essential to make a priori assumptions about the functional form of preferences in order to ensure that necessary conditions for the parameters of the model are met. This limits the flexibility of the model, makes maximum likelihood estimation of the parameters more difficult, and also may place inappropriate restrictions on model outcomes.

In order to deal with these criticisms, a discrete hours approach to estimating the labour supply choice has become popular.

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<sup>3</sup>Note that wages for those not in work are estimated using the Heckman correction in order to deal with selection bias [19]

### 2.3 Discrete hours: van Soest's method

Although having agents optimise their hours of work as a continuous variable appears natural, it comes with the costs mentioned above.

A popular way of getting around this is to have agents choose from a discrete set of potential working hours as shown in van Soest (1995)[43]. Here, our agent  $i$  picks from a discrete choice set of  $j$  income and leisure alternatives,  $\{(y_j, l_j)\}$ . In the same way as in the continuous hours approach,  $\beta$  and  $\epsilon_i$  are estimated given the assumption that the agent is picking the level of hours  $h$  that is optimal.

Creedy and Kalb (2005)[5] discusses the process of estimation in more detail<sup>4</sup>. The agent is assumed to maximise their utility with error. This implies that they choose  $h$  that maximises expected utility where<sup>5</sup>:

$$u_j^* = u_j(h_j|z_i) + v_j \quad (4)$$

Here  $v_j$  could be due to mismeasurement of characteristics, unobserved characteristics, or optimization errors by agents. Given our characterisation of the agents choice, we want to generate a probability distribution for labour supply.

Equation 4 describes a distribution of utility for each discrete hours level, depending on the distribution of  $v_j$ . Utility maximisation implies that for  $n$  discrete hours levels, where  $j, g \in n$ ,  $u_j^*$  is chosen when  $u_j^* \geq u_g^* \forall g$ . Replacing these terms with the deterministic utility component and the stochastic error, we can say that for  $v_j$  the probability of  $j$  being chosen is the joint probability of  $v_g \leq v_j + u_j - u_g$  over all  $g$ . Assuming the various distributions are independent gives the conditional probability for a given value of  $v_j$ :

$$\prod_{g \neq j} P(v_g \leq v_j + u_j - u_g) \quad (5)$$

As this provides the conditional probability for a single draw of  $v_j$ , we then find the full probability of the  $j$ th hours level being chosen by summing this probability over all possible values of  $v_j$ .

If  $v_j$  takes only a discrete values,  $a_k$  for  $k = 1, \dots, K$  we can represent  $p_j$  (the probability  $h_j$  is chosen) as:

$$p_j = \sum_{k=1}^K [\prod_{g \neq j} F(a_k + u_j - u_g)] f(a_k) \quad (6)$$

A numerical example of this is given in Creedy and Kalb (2005).

<sup>4</sup>A significantly more detailed discussion with reference to the Melbourne model MITTS can be found in the book Creedy et al. (2002)[6].

<sup>5</sup>Note, utility is a positive function of consumption  $c$  and a negative function of hours worked  $h$ . However, through the budget constraint hours worked determines income, which determines consumption, therefore  $c(h)$ . This notation simply suppresses this. Eg,  $c = f(hw, y_0)$  where  $f$  is a function that transforms nonlabour income and gross labour income into after-tax agent income. Furthermore,  $y_{0i}$  is included in the individual characteristics  $z_i$ .

When  $v$  is a continuous random variable (although the hours choice continues to be discrete) this sum becomes:

$$p_i = \int_{-\infty}^{+\infty} [\prod_{g \neq j} F(v_j + u_j - u_g)] f(v_j) dv_j \quad (7)$$

The functional form of  $v$  is often assumed to be of the extreme value distribution<sup>6</sup>. This involves taking the density function:

$$f(v) = e^{-v} e^{-e^{-v}} = \exp(-v - e^{-v}) \quad (8)$$

Which gives the distribution function:

$$F(v) = e^{-e^{-v}} \quad (9)$$

This allows us to rewrite  $p_j$  as:

$$p_j = \frac{e^{u_j}}{\sum_{g=1}^n e^{u_g}} \quad (10)$$

Given this structure, the microdata is then used to estimate the unobserved parameters of the model, the preference parameters that make up our unobserved utility<sup>7</sup>. In the discrete hours approach, we can estimate our result over any legitimate utility function. Once given a form for the utility function, we use MLE over our previously defined probability function to estimate the parameters of interest.

Note: There are significant complications involved in the type of agent that is chosen, both in terms of the method used and the interpretation of results. Within families there is joint decision making regarding labour supply and consumption, even though the observed labour supply is tied to an individual. These specific issues will be put to the side here.

### 2.3.1 Moving into the simulation

The prior discussion focused on the specification of the labour supply model, and touched on estimation of the relevant parameters. We now want to use the estimated parameters within this specification to simulate the impact of policy changes on incomes and tax revenue, given a labour supply response by our agents.

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<sup>6</sup>Where more on this can be found in Maddala (1983)[30]. This function holds the favourable property of *independence from irrelevant alternatives*, discussed in Dagsvik and Jia (2008)[7] as the assumption of probabilistic rationality.

<sup>7</sup>Note that I've shown the probability of an hours choice for one individual, but when we estimate we want to use information from the cross-section of individuals - so we want the joint probability of a set of hours. Assuming that decisions around hours are made independently by agents, this is the product of the individual probabilities

Starting off we want a *base period data set* which captures this information based on current policy. We have observed labour supply outcomes and a particular tax-benefit system. We can then use calibration to force the theoretical structure for utility maximising agents to match our *stylized facts* (in this case the labour supply outcomes given the structure of the tax-benefit system). As the deterministic parameters are fixed from estimation, calibration takes place in *errors*,  $v_j$ .

As a result, calibration involves drawing error terms from a given distribution (eg the extreme value distribution) and then adding them to utility for each hours point. If this draw is such that the observed labour supply is the optimal choice ( $\max u^*$ ) the draw is accepted. If not another set of error terms is drawn and checked. This process is repeated until we have the required number of error terms.

Given the draws of error terms, and information on agents characteristics, it is then possible to simulate how utility levels (and thereby choices of  $h$ ) change as policy parameters are changed. This allows us to calculate a post-reform distribution of labour supply.

## 2.4 Discrete hours: Dagsvik and job choice

With both continuous hours and the Van Soest discrete hours approach, the BMSM functioned as a partial equilibrium model, with only a labour supply response to changes in tax-benefit policy. However, as we noted there are a variety of general equilibrium effects that have been assumed away as simplifying assumptions.

Specifically, Dagsvik suggests that it is possible to include agents *preferences* over qualitative job-specific factors or *choice restrictions* facing the agent in the labor market due to a restricted choice set of job opportunities Dagsvik and Jia (2008)[7]. This was first suggested in Dagsvik and Strom (1988)[9] and published in its current form in Dagsvik and Strom (2006)[10].

He suggests modelling labour supply decisions as if workers face a latent workers-specific choice set of jobs (where a job is made up of a fixed combination of hours, wages rate, and non-pecuniary attributes). In this case the agent specific restriction on job opportunities, and the attributes of a chosen job, are additional sources of unobserved heterogeneity in a traditional model.

The practical relevance of this choice is also justified by how poorly the initial version of the van Soest fits with the observed peaks in hours for both full time and part time work - an empirical issue that has led to the introduction of ad hoc adjustments in practice [42]. Dagsvik and Strom suggest that including job opportunities can deal with this shortcoming.

Note: There are significant conceptual and practical complications involving representing and measuring qualitative job types given observed characteristics and outcomes ( $h$  and  $w$ ), but these will be put to the side for now.

Given the idea that people are choosing jobs, the general utility function can be rewritten as:

$$U(c, h, a) = \nu(c, h)\epsilon(a) \quad (11)$$

Where  $a$  refers to market opportunities (jobs)  $a = 1, 2, \dots$  and  $a = 0$  refers to a non-market alternative<sup>8</sup>. For each job, hours of work and the net wage are assumed to be fixed at  $(H(a), W(a))$ .

Furthermore  $\epsilon(a)$  is a random taste shifter, which accounts for unobservable individual characteristics and nonpecuniary job-type attributes. The  $\epsilon(a)$  is generally assumed to take the form of the extreme value distribution.

Given the budget constraint  $c = f(hw, y_0)$  we can define a *representative utility of jobs* function for the  $i$ th individual as:

$$\psi(h, w, y_0) \equiv \nu(f(hw, y_0), h) \quad (12)$$

There is a restricted set of market opportunities (jobs) available to a given worker, a set that is unobservable.  $B(h, w)$  denotes the agents available set of jobs which contains  $H(a) = h$  and  $W(a) = w$ .  $m(h, w)$  is the number of jobs in  $B(h, w)$  while there is one non-market alternative  $m(0, 0) = 1$ .

Given  $D$  as the set of all possible hours of work, and  $G$  as the set of possible values of the wage rate, and given our assumptions about  $\epsilon(a)$  we can say that the probability that a specific job,  $a$ , is chosen is:

$$P\left(\psi(h, w, y_0)\epsilon(a) = \max_{x \in D, y \in G} \max_{k \in B(x, y)} (\psi(x, y, y_0)\epsilon(k))\right) = \frac{\psi(h, w, y_0)}{\sum \sum_{x \in D, y \in G, k \in B(x, y)} \psi(x, y, y_0)} \quad (13)$$

Which is equal to:

$$\frac{\psi(h, w, y_0)}{\psi(0, 0, y_0) + \sum_{x \in D, x > 0} \sum_{y \in G} \psi(x, y, y_0)m(x, y)} \quad (14)$$

$\varphi(h, w|y_0)$  denotes the probability that the agent chooses a job with hours  $h$ , wage rate  $w$ , and individual non-labour income  $y_0$ . This is equal to the probability of any job within  $B(h, w)$  so involves summing the probabilities of choosing each individual job  $a$  within  $B(h, w)$ . For  $h, w > 0$  this is:

$$\varphi(h, w|y_0) = \frac{\psi(h, w, y_0)m(h, w)}{\psi(0, 0, y_0) + \sum_{y \in G} \sum_{x > 0, x \in D} \psi(x, y, y_0)m(x, y)} \quad (15)$$

And for  $h = 0$

$$\varphi(0, 0|y_0) = \frac{\psi(0, 0, y_0)}{\psi(0, 0, y_0) + \sum_{y \in G} \sum_{x > 0, x \in D} \psi(x, y, y_0)m(x, y)} \quad (16)$$

<sup>8</sup>Note that it is common to assume that the structural and random term are uncorrelated such that  $U(c, h, a) = \nu(c, h) + \epsilon(a)$  [11]



Both  $m(h, w)$  and  $\psi(h, w, y_0)$  are unobservable, and so we have to impose a set of assumptions in order to estimate their parameters. Dagsvik and Jia (2008) goes on to discuss this process, and how to make the latent choice sets of jobs differ across agents. Identifying  $m(h, w)$ , and estimation of parameters, is discussed in more detail in Dagsvik and Jia (2012)[8].

A common decomposition of the market opportunities term is that  $m(h, w) = \hat{\theta} \hat{g}(h, w)$ . Here  $\hat{\theta} = \sum_{y \in G} \sum_{x \in D, x > 0} m(x, y)$  is a *opportunity measure* (total number of jobs available to the agent). And  $\hat{g}(h, w) = m(h, w) / \hat{\theta}$  is the *opportunity distribution* (fraction of jobs available that have the combination  $(h, w)$ ).

In Dagsvik et al. (2011)[11] the difference between the van Soest and this approach is given in terms of the conditional probability of choosing  $h$  given  $w$ . Namely, given the assumption that the errors follow the extreme value distribution, the van Soest approach yields:

$$p(h|w, z) = \frac{\psi(h, w)}{\psi(0, 0) + \sum_{x \in D} \psi(x, w)} \quad (17)$$

While the [11] measure involves  $(h, w)$  being codetermined, so is:

$$p(h, w|a, z) = \frac{\psi(h, w)m(h, w)}{\psi(0, 0) + \sum_{x \in D} \sum_{y \in G} \psi(x, y)m(x, y)} \quad (18)$$

This implies that the van Soest result is equivalent to the Dagsvik and Strom (2006) result when the opportunity measure is independent of  $h$  and equal to zero for  $w$  different from the industry specific wage rate, and one otherwise.

As a result of this correspondence, Dagsvik et al. states that the van Soest result is a subset of the general job opportunity result with more restrictive assumptions - essentially the van Soest result is equivalent to assuming that job availability constitutes a set of jobs with varying hours rates at a given gross wage.

Thoresen et al. (2013)[42] uses this framework to evaluate Norweign tax policy between 2000-2010.

### 3 CGE: Labour Demand and other margins

As we noted at the start, there is a distinct trade-off between the heterogeneous agent approach associated with microsimulation modelling, and the more behaviourally focused approach associated with representative agent modelling.

Although including labour supply responses, and introducing a measure of *job opportunity* help to bridge some of the gap between the modelling techniques, there are still significant areas of divergence.

Spadaro (2007) points out three general ways that microsimulation modelling and macro/representative agent modelling can be tied together to improve the usefulness of both.

- The estimation of a wage equation that is a function of household/individual (endowed) characteristics<sup>9</sup>.
- The estimation of a combined CGE-MSM model, with the representative agents in a CGE model replaced by a greater number of agents based on household microdata.
- The sequential estimation of a CGE and MSM model, with either *top-down* or *bottom-up* linkages (eg Verikios and Zhang (2013)[44]).

A key area where a CGE model can add value in microsimulation work is by allowing the analysis of *third round* effects. Furthermore, with the inclusion of intertemporal effects the use of CGE would allow for broader and richer changes in the structure of hiring, capital investment, and activity across the economy to be estimated.

By accounting for changes in the level of gross wages, and the relative structure of gross wages across occupations/industries, this also rounds off the labour market story and allows for a more realistic view of tax incidence - which the prior MSM's treated as if they fell solely on the modelled agent.

### 3.1 Adding components to a microsimulation model from within

Creedy and Duncan (2005)[4] focuses specifically on estimating *third round* effects in the analysis of tax reform from within a microsimulation model.

As mentioned above, when looking at tax reform the easing of assumptions allows us to progressively move towards a more detailed view of tax incidence involving utility maximising agents when using a MSM. As a result, we can use the theory of incidence to help inform the process.

Following Spadaro (2007), our agents indirect utility function takes the form:

$$V_i(p, y_i) = \{\max\{U_i(x_i) \text{ st } x_i p \leq y_i\}\} = U_i(x_i^M(p, y_i)) \quad (19)$$

Here  $y = hw + y_0 - NT(h, w, y_0; \gamma)$ . Following a tax change, we assume that the price vector of goods and services remains fixed<sup>10</sup>. Given that assumption, we have  $\Delta V = V_y \Delta y$  where  $V_y$  can be normalised to one without loss of generality. In that case, it is the change in our budget constraint due to the policy change,

<sup>9</sup>Note that these types of methods are diverse and can go from just estimating a Mincer type wage equation [Juhn et al. (1993)[25], DiNardo et al. (1996)[13], Hyslop and Yahanpath (2005)[23]] to simulating household income [Hyslop and Mare (2005)[22]]

<sup>10</sup>For a very large policy change, this assumption would not hold.

and what that descriptively implies for the outcome of hours and wages, that we are interested in modelling<sup>11</sup>.

An AMSM changes the function  $NT$  but leaves  $h, w$ , and  $y_0$  fixed. In this case agents bear the full cost of the tax, and do not change their behaviour.

A BMSM allows for changes in the agents choice of  $h$  but leaves the gross wage  $w$  unchanged. In this case, the labour demand curve is essentially being treated as if it is flat (perfectly elastic) in gross wages (the cost to the firm). As a result, tax incidence all falls on the agent that is modelled and the response comes solely through the number of hours of labour supplied.

Modelling the third round response involves considering how firms would actually respond to the estimated change in hours, by adjusting the level and distribution of gross wages.

While the Dagsvik and Strom (2006) method does allow for some wage adjustment, it is within the prior set of job opportunities for the agent. As a result, this is a separate issue to the third round effects we are discussing.

However, aggregating and feeding information into a representative agent macro model is problematic given the difficulty of relating the individual agents wages to a representative wage.

The suggestion in Creedy and Duncan is to take the weighted average of individual labour supply responses from a BMSM. This gives a single point on a labour supply schedule. Then the model is perturbed, meaning that the the response of individual hours to proportional shifts in the wage distribution are taken. This provides additional points, allowing the formation of a supply schedule. Furthermore, each point on the schedule contains information on the distribution of wage, which implies that when it is used for analysis the distributional information is maintained and can be fed back into a microsimulation model.

The schedule is termed a *aggregate supply response schedule*, and it shifts in response to policy. Given that we can consider the implications for wages and hours given differing assumptions about labour demand (which could be sourced from an outside model). Furthermore, it is possible to apply this process to individual groups (on the basis of characteristics) which would in turn allow for changes in the dispersion of wages.

## 3.2 CGE model and microsimulation

The preceding discussion looked exclusively at the derivation of labour supply, with labour demand analysis put to the side. However, there are times when labour demand movements are especially important (such as the discussion of cyclical effects). A characterisation of labour demand where the employer chooses hours and wages, and it is then fed into a microsimulation model is shown in Bargain et al.2010[2]. The purpose of the Bargain et al. paper is to

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<sup>11</sup>Note that this is a positive not normative analysis, so the maximisation of individual utility is a behavioural assumption, rather than a normative goal given policy parameters. That is why it is discussed in this way.

discuss the impact of an economic slowdown, and in this way starting with a model of labour demand makes sense.

More broadly, there are growing numbers of examples where MSM-CGE models are linked to discuss tax and labour reform policy questions. Magnani and Mercenier (2009)[31], Peichl (2008)[38], and Boeters and Feil (2009)[3] all use a discrete choice labour supply framework as the base for their MSM analysis.

The combination of static microsimulation and CGE modelling makes considerable sense, given both frameworks incorporate a comparative static view of the economy and policy questions - but embody different sets of assumptions when developing their counterfactual world.

More broadly, Ahmed and O'Donoghue (2008)[1] and Estrades (2013)[14] provide surveys of ways MSM-CGE models can be linked. A recent example of these linkages applied to Australian public transport is Verikios and Zhang (2013).

A brief, but clear and concise, summary on the types of linkages used in practice can be found in Lay (2010)[26].

## 4 Ageing: Dynamic vs Static Microsimulation

The above description was entirely based on static snapshot data. However, if we want to compare two points in time there are two types of *ageing* we can consider [34]:

- **Dynamic ageing** works by changing the characteristics of the micro-units in response to accumulated experience or the passage of time.
- **Static ageing** uses a combination of re-weighting of micro-units (by characteristics) and indexation of money amounts to update cross-sectional micro-data to the required point in time.

Dynamic ageing models are based on longitudinal data<sup>12</sup>, so individuals are linked through time. This implies we can incorporate changes in the *stock* of given variables and characteristics, as well as estimating transition probability matrices that represent probability of the movement between *states of the world* through time.

Static ageing models are based on cross-sectional static snapshots, so we cannot identify the same individual in different snapshots. Static ageing involves assuming that shared characteristics tell us enough about individuals, and that the change in the characteristics is exogenous, such that we can reweight and compare characteristics through time. As changes in characteristics are likely to be endogenous (as they can be viewed as a stock of a given characteris-

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<sup>12</sup>This is in an ideal world, as noted in [27] - there are papers that suggest that dynamic ageing is also incorporating transition probabilities into pooled cross sectional data eg [34],[16]

tic), this weakness in estimation needs to be recognised when using static models<sup>13</sup>.

Static ageing is still very useful for describing changes, however the fact we can't link agents through time makes any causal claims weaker than in the case of dynamic ageing.

A survey of dynamic microsimulation modelling (DMSM), including a sizable appendix discussing the details of different in use DMSM's, can be found in O'Donoghue (2001)[35]. A more up-to-date version can be found in O'Donoghue and Li (2012)[36].

#### 4.1 State-dependent Intertemporal Labour Supply with Panel data

Jia and Vattø (2013)[24] suggest that an intertemporal flavour for labour supply adjustments, incorporating state dependence, can be incorporated into BMSMs using the Dagsvik and Strom (2006) method of job choice.

The model of Hyslop (1999)[21] is used to justify the search for an intertemporal model of labour supply with state dependence using longitudinal data. Jia and Vattø extends this method to a microsimulation context using panel data.

In this paper, they state that:

The idea in our framework is that the previous choice of working hours influence both current preferences for leisure and consumption, in addition to current job opportunities.

Essentially, they model agents that make a myopic choice regarding  $h$ . This involves no consideration of key two intertemporal factors:

- Lifetime income smoothing: Characteristics such as age are included, and will in turn have an impact on the decision of how many hours to work. However, an intertemporal lifetime budget constraint is not taken into consideration, implying that these matters are only dealt with indirectly.
- Intertemporal optimisation via job choice: Agents do not take into account how their job choice will influence future costs and benefits.

The authors point out that such intertemporal optimisation add significant complications, and that as a simplifying assumption the assumption of myopia has a place. The key result of Jia and Vattø is that there is an adjustment period from when a policy is put in place until the point in time when the *static* result holds. Their method allows us to quantify and describe this transition period.

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<sup>13</sup>Although I have been told pooled cross-sectional methods may be useful (by creating psuedo-panels, eg Deaton1985[12]), this still does not link individuals through time [45]. So far, I have also not found anything that suggests this is practically different to reweighting cross-sectional survey data - as when the characteristics are exogenous, eg age, both methods will give similar conclusions. But when they are endogenous they merely look like competing explanations of transition probabilities for those characteristics, which implies that this is the specific focus

Termining the *static* result as the *long-run* result<sup>14</sup> the paper notes that their results converge to the static result. However, given the agents are myopic this result is to be expected, as it stems from the stability of the equilibrium. If agents made choices on the basis of these future costs, and these costs varied by job type, the equilibrium would likely change<sup>15</sup>. As a result, this convergence can be seen as illustrating that their results are consistent with the general literature. The key contribution is in showing how this adjustment process may take place.

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<sup>14</sup>Although we have to be careful here, as in the static model does not model both the stock of, and rate of accumulation of, physical and human capital. So this is not a long-run, or even medium run (when prices and relative variables such as rates of accumulation have changed - but stocks are still adjusting), in the traditional sense

<sup>15</sup>This can be seen from looking at the appropriate Bellman equation for a myopic vs a forward looking agent,  $V_t(h) = U_t(h)$  vs  $V_t(h) = U_t(h) + \delta V_{t+1}(h)$ , thereby implying the lack of any potential intertemporal trade-off in the current specification

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